McKeesport Area School District

Flexible Instruction Days – High School Lesson Plan

SUBJECT: Algebra 2			**LESSON TITLE:** Simplifying Rational Expressions.	
☐**LESSON 1:** **Factoring the GCF.** 1st or 2nd 9-Weeks	☐ **LESSON 2:** **Factoring quadratic equations.** 2nd or 3rd 9-Weeks	☐ **LESSON 3:** **Solve equations by factoring.** 2nd or 3rd 9-Weeks	☐ **LESSON 4:** **Graphing quadratic equations.** 2nd or 3rd 9-Weeks	☒ **LESSON 5:** **Simplifying rational expressions.** 3rd or 4th 9-Weeks
STANDARD(S): CC.2.2.HS.D.1, CC.2.2.HS.D.3				

INSTRUCTIONAL OUTCOMES:

Students will:

- Apply factoring techniques to write expressions in equivalent forms.
- Simplify rational expressions using multiplication/division.

STUDENT PARTICIPATION *(Lesson steps):*

Students will:

1. Study the examples found on each Study Guide.
2. Use the Factoring Techniques page to aid in the factoring process.
3. Complete Study Guide 9.1 pg. 113: Simplify rational expressions using multiplication/division.
4. Complete Study Guide 9.1 pg. 114: Simplify rational expressions using multiplication/division.

ACCOMMODATIONS:

For struggling learners:

- **Use the perfect squares chart.**
- **Reduce the required number of problems to odd problems only.**
- **Use a calculator to aid in identifying possible factors.**
- **Use the multiplication table to aid in identifying possible factors.**

For advanced learners:

- Students must complete all problems on both Study Guides.
- Choose one problem, and justify each step in the simplification process.

HANDOUTS *(exact names of ALL accompanying handouts)* **& RESOURCES** *(materials, websites, books, etc.)*

- Calculators
- Perfect squares chart
- Multiplication table
- Study Guide 9.1 pg. 113, Study Guide 9.1 pg. 114
- Important Factoring Techniques
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:cancel-common-factor/v/simplifying-rational-expressions-introduction
- https://www.youtube.com/watch?v=RROSgr4oXjU
- https://www.youtube.com/watch?v=rMVOdIDNjpA

EVIDENCE OF LEARNING

Students will demonstrate their:

- Understanding of how to apply factoring techniques to simplify rational expressions.
- Understanding of how to apply the rules of multiplying and dividing rational expressions.
- Understanding of how to write expressions in equivalent forms.

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9-1 Study Guide and Intervention

Multiplying and Dividing Rational Expressions

Simplify Rational Expressions A ratio of two polynomial expressions is a **rational expression**. To simplify a rational expression, divide both the numerator and the denominator by their greatest common factor (GCF).

Multiplying Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.
Dividing Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Example **Simplify each expression.**

a. $\dfrac{24a^5b^2}{(2ab)^4}$

$$\frac{24a^5b^2}{(2ab)^4} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3 \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot a \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot b \cdot b} = \frac{3a}{2b^2}$$

b. $\dfrac{3r^2n^3}{5t^4} \cdot \dfrac{20t^2}{9r^3n}$

$$\frac{3r^2n^3}{5t^4} \cdot \frac{20t^2}{9r^3n} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{r}} \cdot \overset{1}{\cancel{r}} \cdot \overset{1}{\cancel{n}} \cdot n \cdot n \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{t}} \cdot \overset{1}{\cancel{t}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{r}} \cdot \underset{1}{\cancel{r}} \cdot t \cdot t \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot \underset{1}{\cancel{r}} \cdot \underset{1}{\cancel{r}} \cdot r \cdot \underset{1}{\cancel{n}}} = \frac{2 \cdot 2 \cdot n \cdot n}{3 \cdot r \cdot t \cdot t} = \frac{4n^2}{3rt^2}$$

c. $\dfrac{x^2 + 8x + 16}{2x - 2} \div \dfrac{x^2 + 2x - 8}{x - 1}$

$$\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1} = \frac{x^2 + 8x + 16}{2x - 2} \cdot \frac{x - 1}{x^2 + 2x - 8}$$

$$= \frac{\overset{1}{\cancel{(x + 4)}} (x + 4) \overset{1}{\cancel{(x - 1)}}}{\underset{1}{\cancel{2(x - 1)}} (x - 2) \underset{1}{\cancel{(x + 4)}}} = \frac{x + 4}{2(x - 2)}$$

Exercises

Simplify each expression.

1. $\dfrac{(-2ab^2)^3}{20ab^4}$

2. $\dfrac{4x - 12x + 9}{9 - 6x}$

3. $\dfrac{x^2 + x - 6}{x^2 - 6x - 27}$

4. $\dfrac{3m^3 - 3m}{6m^4} \cdot \dfrac{4m^5}{m + 1}$

5. $\dfrac{c^2 - 3c}{c^2 - 25} \cdot \dfrac{c^2 + 4c - 5}{c^2 - 4c + 3}$

6. $\dfrac{(m - 3)^2}{m^2 - 6m + 9} \cdot \dfrac{m^3 - 9m}{m^2 - 9}$

7. $\dfrac{6xy^4}{25z^3} \div \dfrac{18xz^2}{5y}$

8. $\dfrac{16p^2 - 8p + 1}{14p^4} \div \dfrac{4p^2 + 7p - 2}{7p^5}$

9. $\dfrac{2m - 1}{m^2 - 3m - 10} \div \dfrac{4m^2 - 1}{4m + 8}$

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9-1 Study Guide and Intervention *(continued)*

Multiplying and Dividing Rational Expressions

Simplify Complex Fractions A **complex fraction** is a rational expression with a numerator and/or denominator that is also a rational expression. To simplify a complex fraction, first rewrite it as a division problem.

Example Simplify $\dfrac{\frac{3n-1}{n}}{\frac{3n^2+8n-3}{n^4}}$.

$$\frac{\frac{3n-1}{n}}{\frac{3n^2+8n-3}{n^4}} = \frac{3n-1}{n} \div \frac{3n^2+8n-3}{n^4}$$ Express as a division problem.

$$= \frac{3n-1}{n} \cdot \frac{n^4}{3n^2+8n-3}$$ Multiply by the reciprocal of the divisor.

$$= \frac{(3n-1)n^4}{n(3n-1)(n+3)}$$ Factor and eliminate.

$$= \frac{n^3}{n+3}$$ Simplify.

Exercises

Simplify each expression.

1. $\dfrac{\frac{x^3y^2z}{a^2b^2}}{\frac{a^3x^2y}{b^2}}$

2. $\dfrac{\frac{a^2bc^3}{x^2y^2}}{\frac{ab^2}{c^4x^2y}}$

3. $\dfrac{\frac{b^2-1}{3b+2}}{\frac{b+1}{3b^2-b-2}}$

4. $\dfrac{\frac{b^2-100}{b^2}}{\frac{3b^2-31b+10}{2b}}$

5. $\dfrac{\frac{x-4}{x^2+6x+9}}{\frac{x^2-2x-8}{3+x}}$

6. $\dfrac{\frac{a^2-16}{a+2}}{\frac{a^2+3a-4}{a^2+a-2}}$

7. $\dfrac{\frac{2x^2+9x+9}{x+1}}{\frac{10x^2+19x+6}{5x^2+7x+2}}$

8. $\dfrac{\frac{b+2}{b^2-6b+8}}{\frac{b^2+b-2}{b^2-16}}$

9. $\dfrac{\frac{x^2-x-2}{x^3+6x^2-x-30}}{\frac{x+1}{x+3}}$

Important Factoring Techniques

Factoring Technique	General Case
GFC	$a^3b^2-nab^2=ab^2(a^2-n)$
General Trinomials	$acx^2+(ad+bc)x+bd=(ax+b)(cx+d)$
Difference of Two Squares	$a^2-b^2=(a+b)(a-b)$
Perfect Square Trinomials	$a^2\pm2ab+b^2=(a\pm b)^2$

Perfect Squares Chart

x^2	Perfect Square
1^2	1
2^2	4
3^2	9
4^2	16
5^2	25
6^2	36
7^2	49
8^2	64
9^2	81
10^2	100
11^2	121
12^2	144
13^2	169
14^2	196
15^2	225
16^2	256
17^2	289
18^2	324
19^2	361
20^2	400

Multiplication Table

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	0	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	0	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	0	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	0	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	0	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	0	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	0	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	0	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625